

# Model Dependence in Counterfactual Inference

Gary King

October 20, 2005

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# References

- King, Gary and Langche Zeng. “The Dangers of Extreme Counterfactuals,” *Political Analysis*, Vol. 14, No. 2, 2006, forthcoming.
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<http://GKing.Harvard.edu>

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- Counterfactuals are part of almost all research questions.

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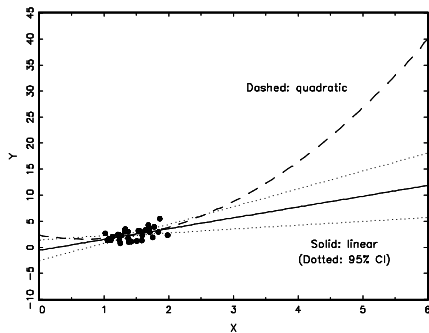
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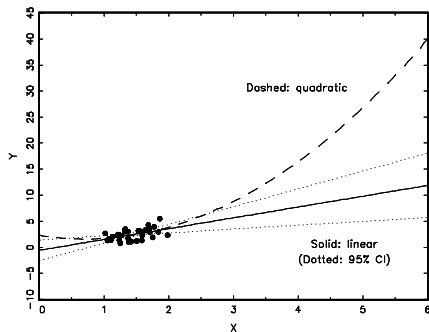
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  - Is this a true test of an ex ante hypothesis or merely a demonstration that it is *possible* to find results consistent with your favorite hypothesis?

# Which model would you choose? (Both fit the data well.)

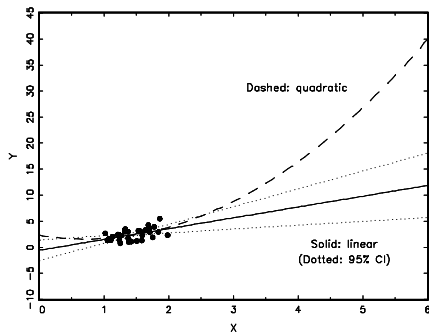


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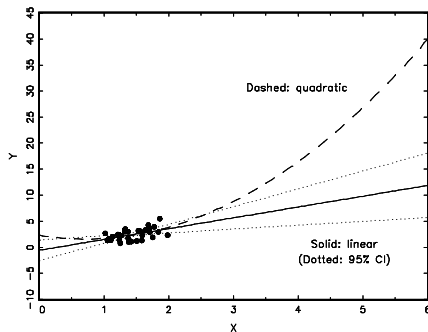
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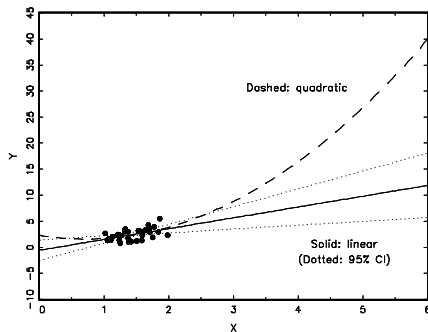
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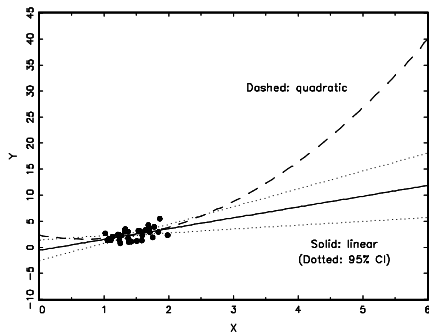
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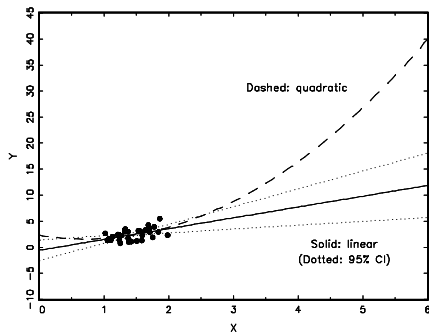
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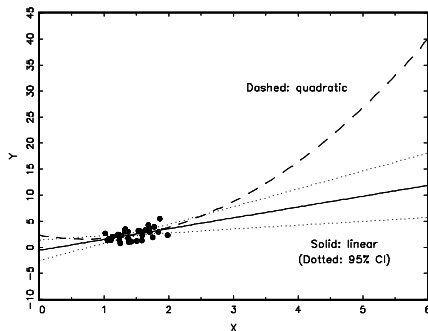
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- How do you choose a model?  $R^2$ ? Some “test”? “Theory”?
- The bottom line: answers to some questions don't exist in the data.
- Same for what if questions, predictions, and causal inferences

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## Result

The maximum degree of model dependence: solely a function of the **distance from the counterfactual to the data**

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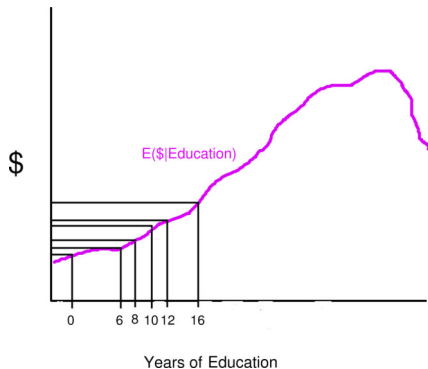
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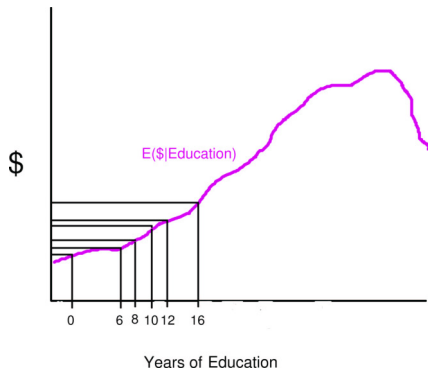
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- We find a coefficient of  $\hat{\beta} = \$1,000$ , big t-statistics, narrow confidence intervals, and pass every test for auto-correlation, fit, normality, linearity, homoskedasticity, etc.

# What Inferences Would You Be Willing to Make?

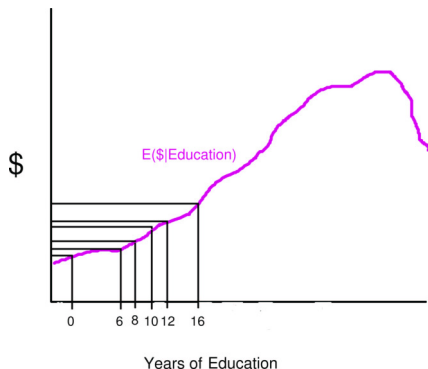


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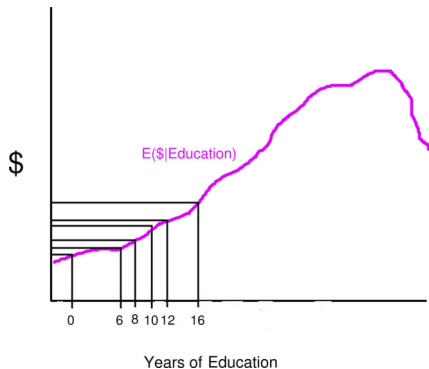
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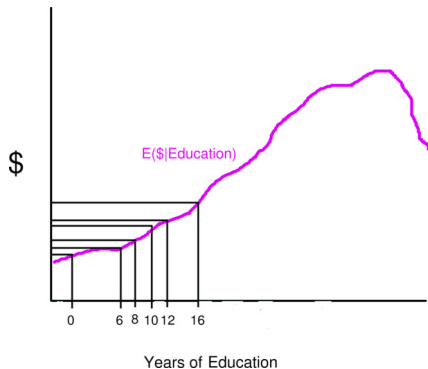
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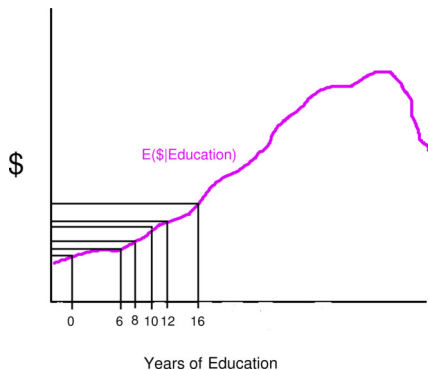


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- The **model-based** linear estimate:  $\hat{Y} = X\hat{\beta} = 12 \times \$1,000 = \$12,000$

# Counterfactual Inferences with Interpolation

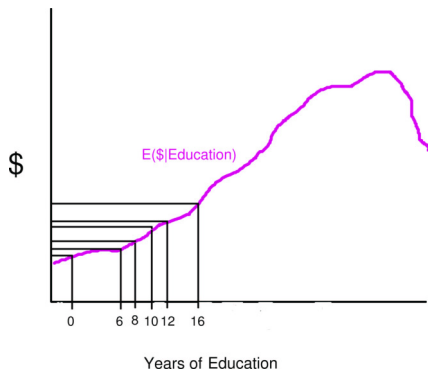


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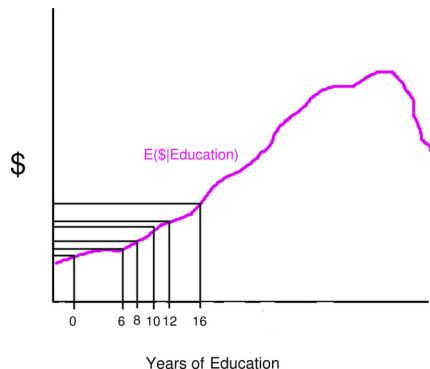
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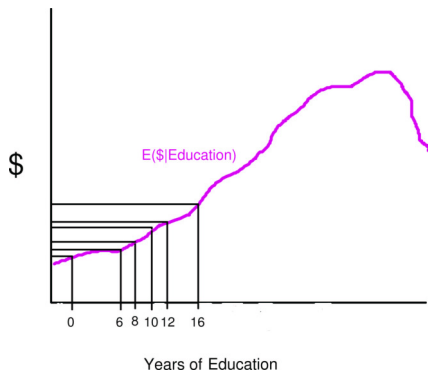
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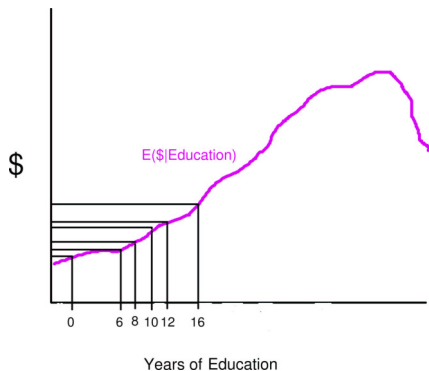


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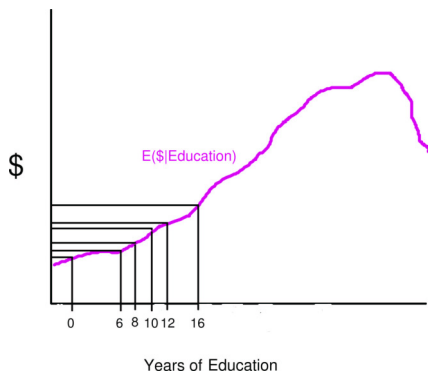


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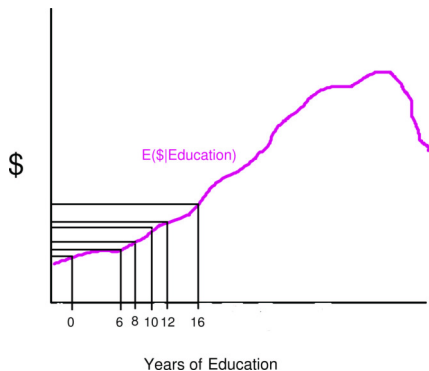
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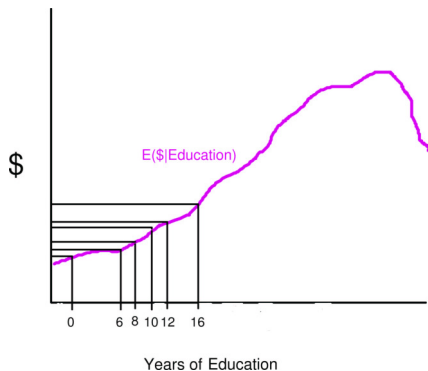


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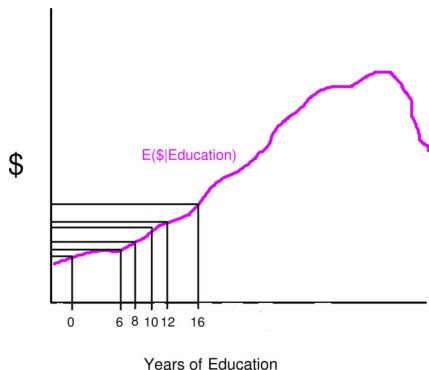


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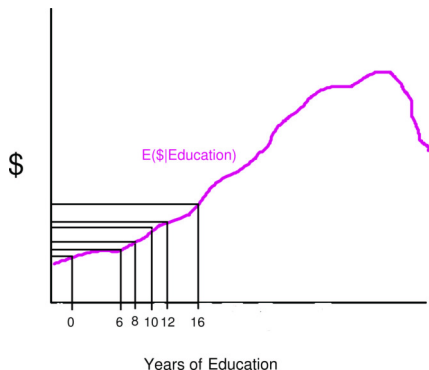
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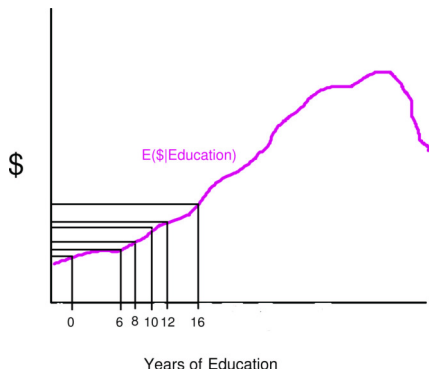
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- What's changed? How would we recognize it when the example is less extreme or multidimensional?

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- If  $X$  were continuous, we would be reducing  $\infty$  to 2, also by assumption.

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- The difference is still one enormous assumption based on convenience, and neither evidence nor theory.

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- Suppose: 15 explanatory variables, with 10 categories each.

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- The curse of dimensionality introduces huge assumptions, often recognized.

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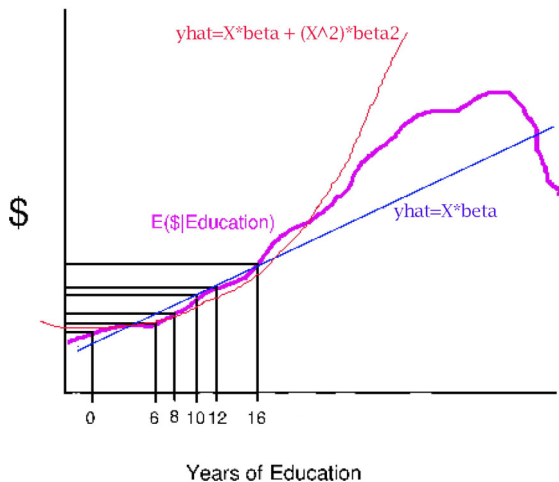
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  - Results of one run apply to the class of all models, all estimators, and all dependent variables.

# Interpolation vs Extrapolation in one Dimension



# Interpolation or Extrapolation in One and Two Dimensions

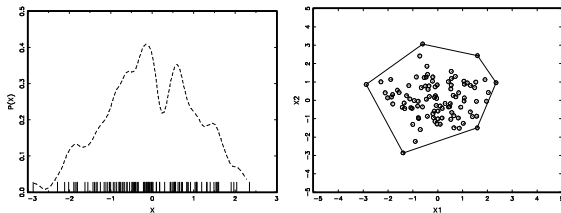


Figure: The Convex Hull

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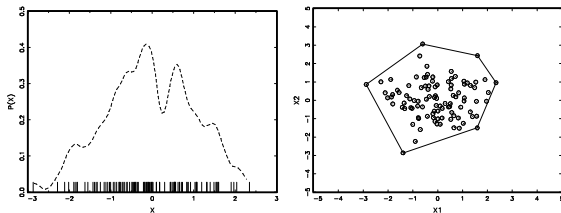


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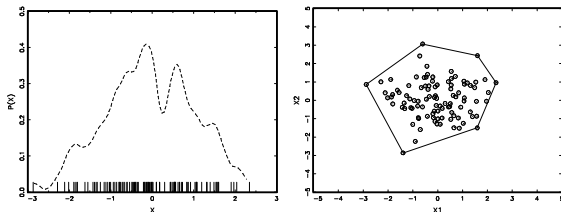


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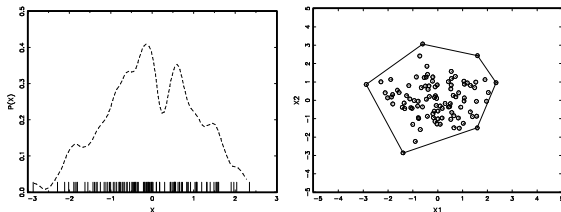


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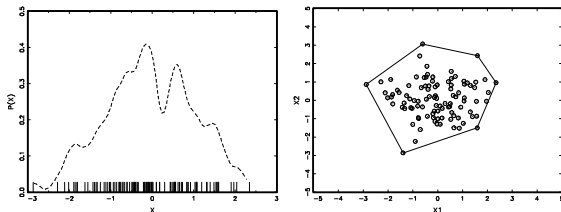


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- We show how to determine whether a point is in the hull without calculating the hull, so its fast; see <http://GKing.harvard.edu/whatif>

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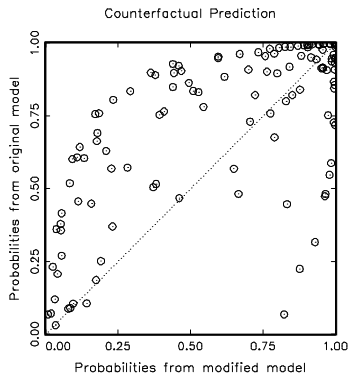
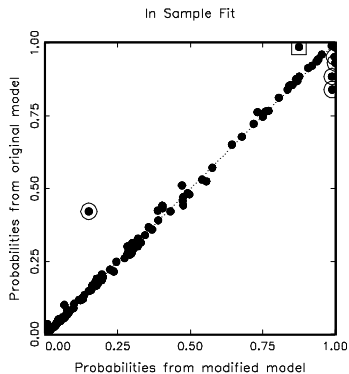
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- Thus, without estimating any models, we know inferences will be model dependent; for illustration, let's find an example. . . .

# Doyle and Sambanis, Logit Model

Variables	Original Model			Modified Model		
	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	-1.742	.609	.004	-1.666	.606	.006
Logdead	-.445	.126	.000	-.437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	-1.259	.703	.073	-1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	-6.016	3.071	.050	-6.215	3.065	.043
Decade	-.299	.169	.077	-0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	—	—	—	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N		122			122	
Log-likelihood		-45.649			-44.902	
Pseudo $R^2$		.423			.433	

# Doyle and Sambanis: Model Dependence



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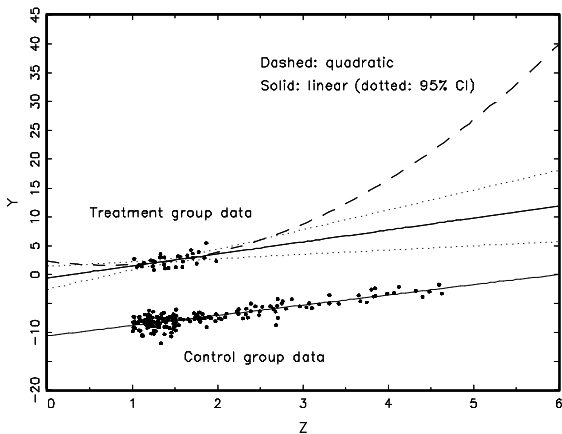
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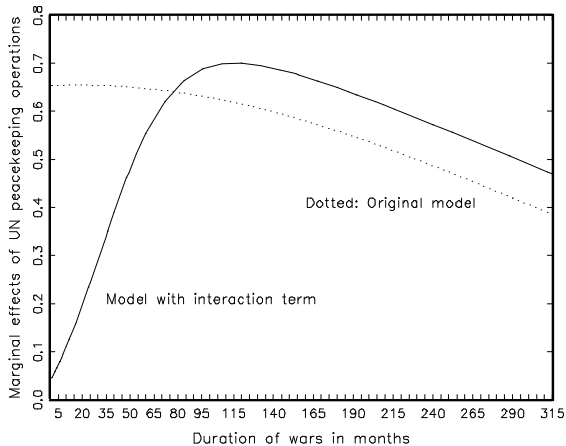
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# Interpolation vs Extrapolation Bias



# Causal Effect of Multidimensional UN Peacekeeping Operations



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  - Most research in every field is observational, and thus requires at least some assumptions.

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- Researchers typically
  - assume a parametric model (up to unknown parameters):  
e.g.,  $Y_i \sim p(\mu_i, \theta)$  with  $\mu_i \equiv E(Y_i | t_i, X_i) = g(\alpha + t_i\beta + X_i\gamma)$
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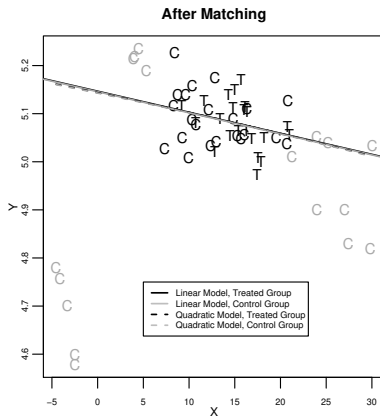
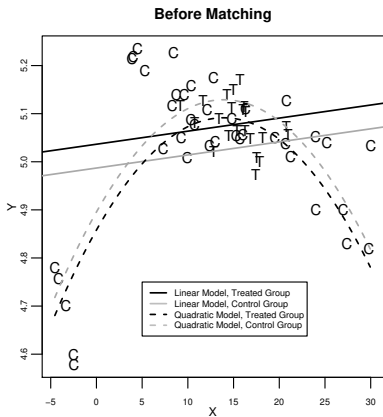
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  - $p(X | t_i = 1) = p(X | t_i = 0)$  or  $p(X | t_i = 1) \approx p(X | t_i = 0)$ .

# A Matching Example



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- Normally, we will only approximate this goal, and will sacrifice some bias reduction (due to lack of balance) for more observations.

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- Parametric Outcome Analysis: same method, same algorithm, same software, same model checking procedures, ...

# Empirical Illustration

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  - 18 control variables (clinical factors, firm characteristics, media variables, etc.)

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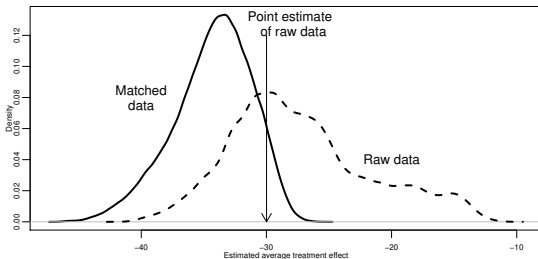
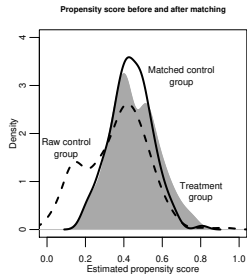
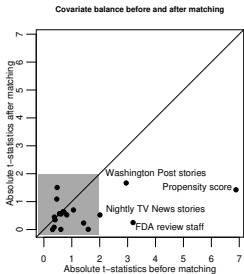
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- (Normal applications would only do one or a small number of specifications.)

# Improved Balance and Reduced Model Dependence



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- Preprocessing the raw data with matching procedures makes familiar parametric models a much more reliable tool.
- Readers (and authors) need not worry that slightly different specifications alter the empirical conclusions.

<http://GKing.Harvard.edu>

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Summarize all the variables in  $X$  with a single variable,  
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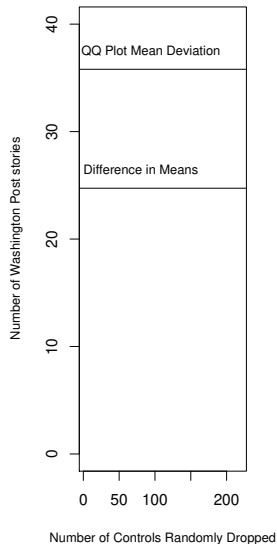
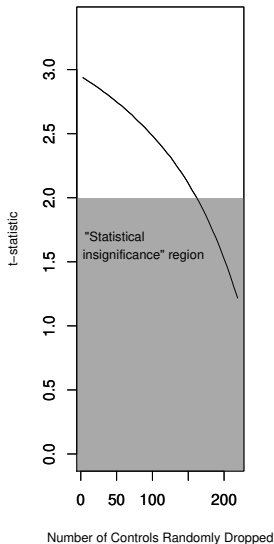
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  - I.e., it works when it works, and when it doesn't work, it doesn't work.

# Hypothesis Tests for Balance Make No Sense



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# Omitted Variable Bias

