Scientific Measurement in Redistricting Research

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The Gerrymandering Project, Princeton University, 5/21/2021

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Principles

Defining and Measuring Compactness

Differential Privacy

Fairness in Redistricting

Requirements for Scientific Measurement

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2. Measure with known statistical properties

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   - Without uncertainty estimates: Measures are useless
   - A scientific statement: not one that is necessarily correct, but one that comes with accurate uncertainty estimates

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  - All know it when they see it & agree on the same dimension

Which is more compact?

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- All agree: Very high intra- and inter-coder reliability

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Respondents ranging from ordinary citizens to those responsible for redistricting

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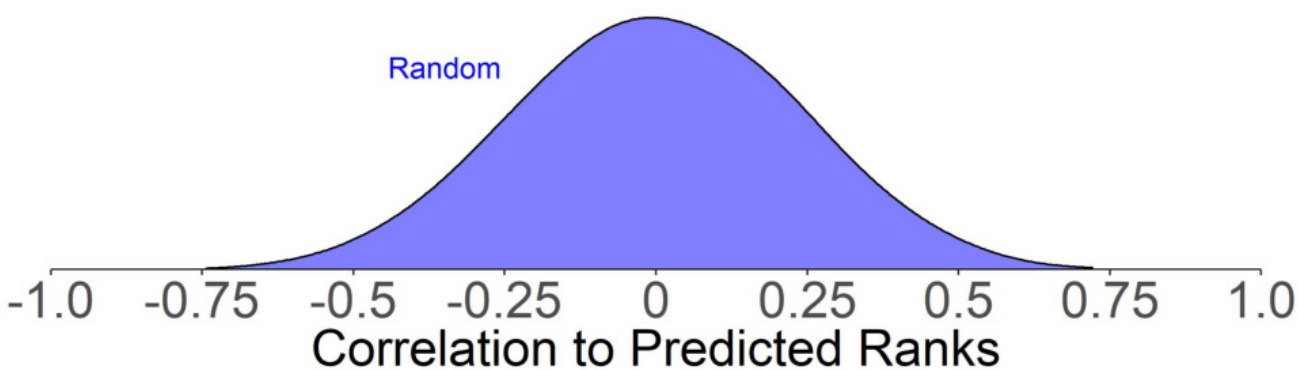

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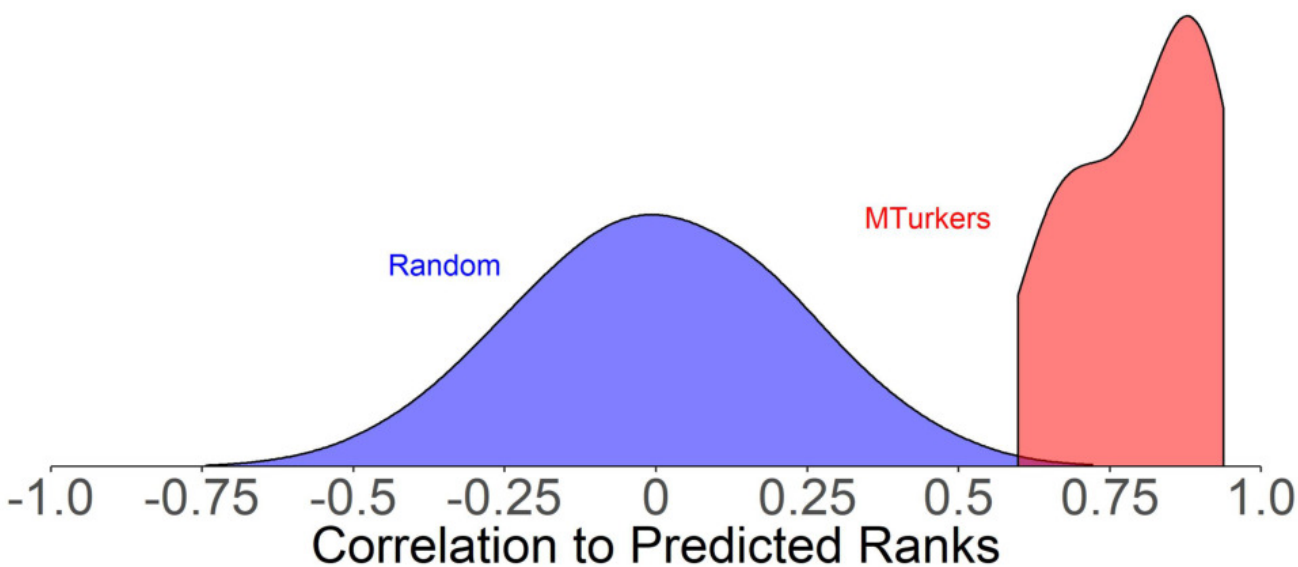

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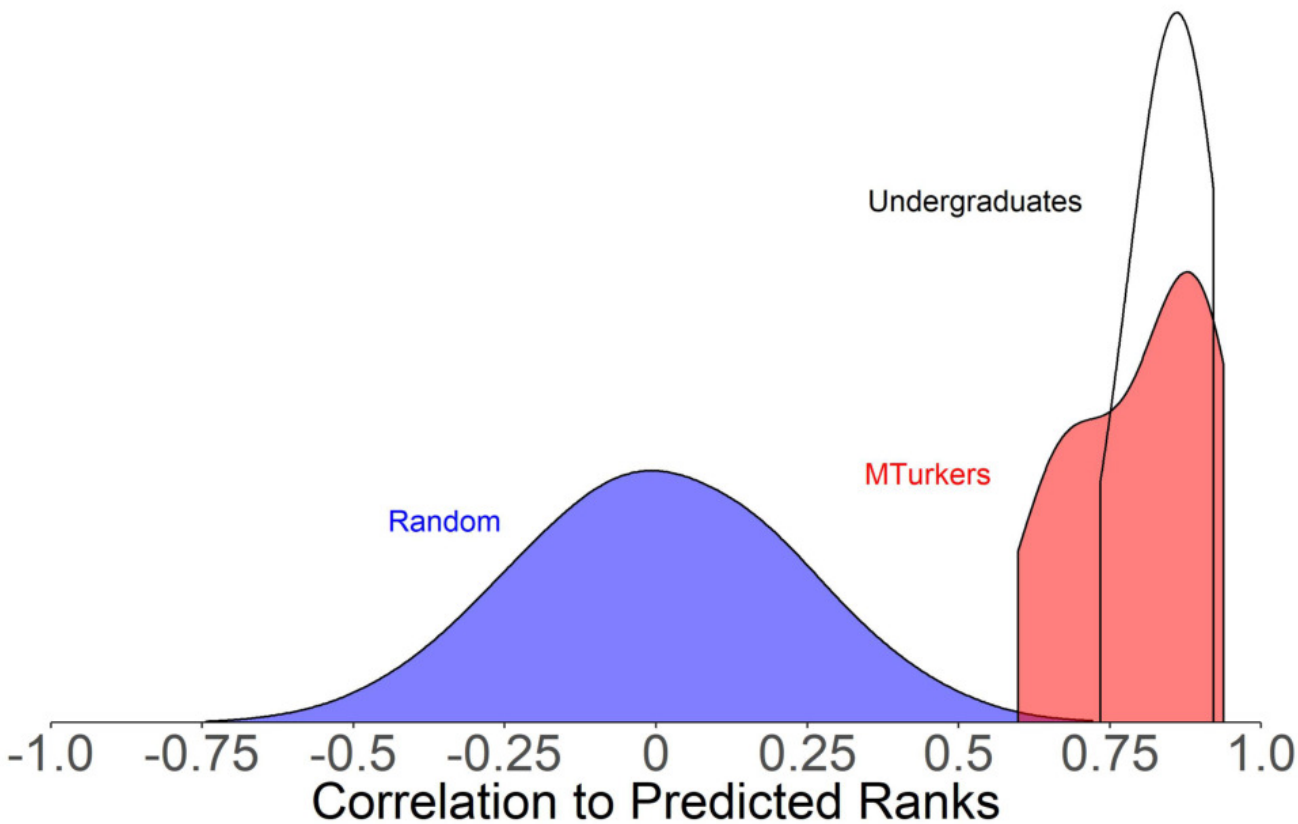

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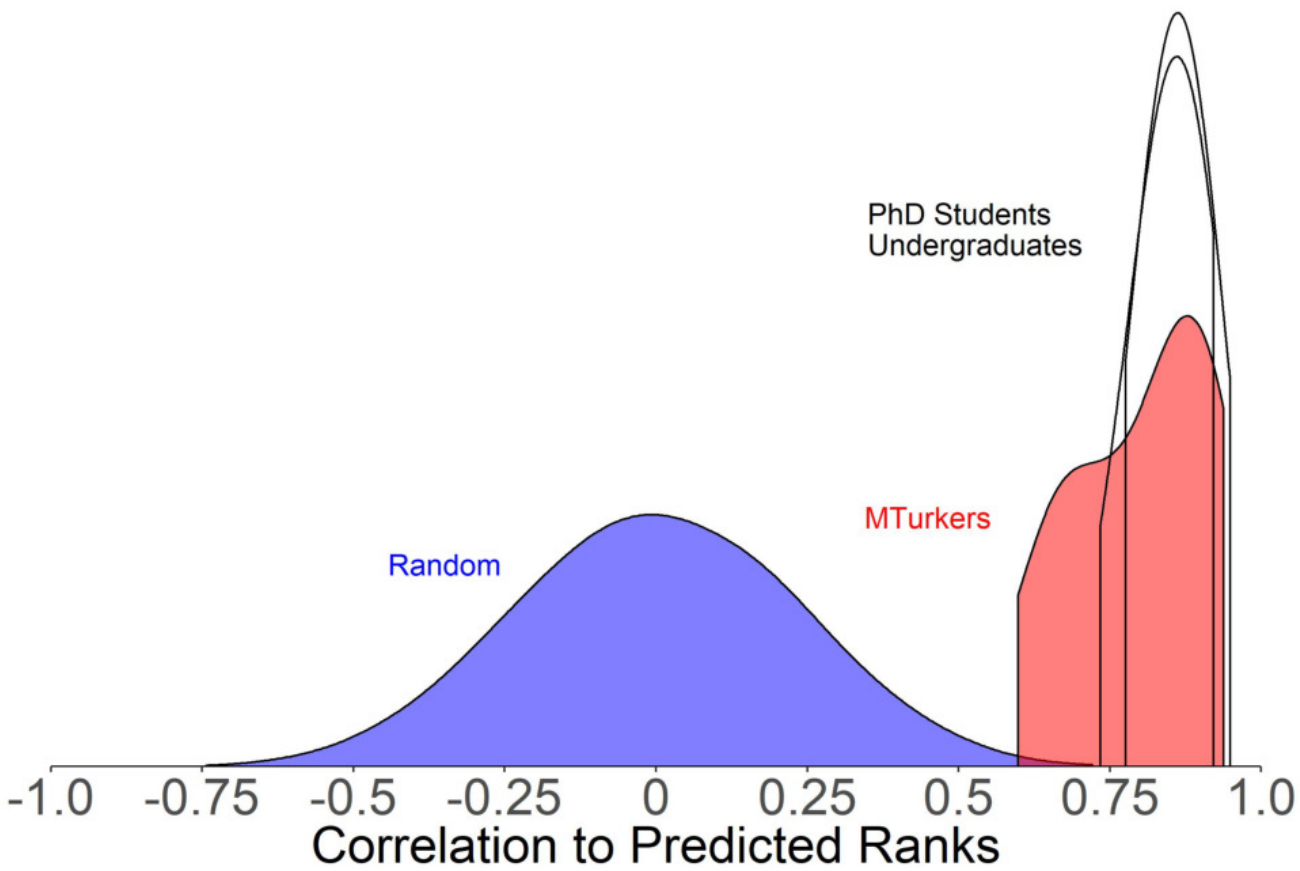

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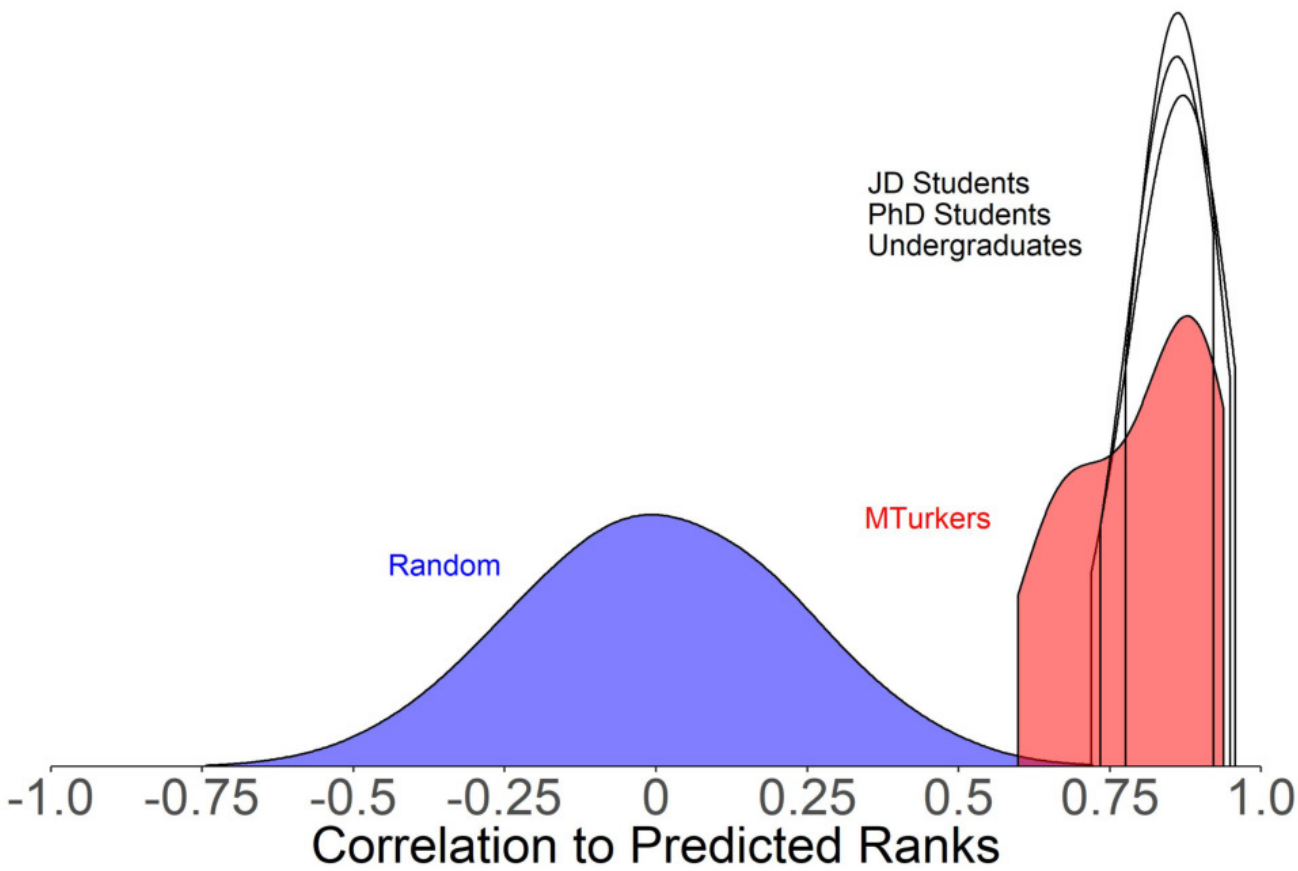

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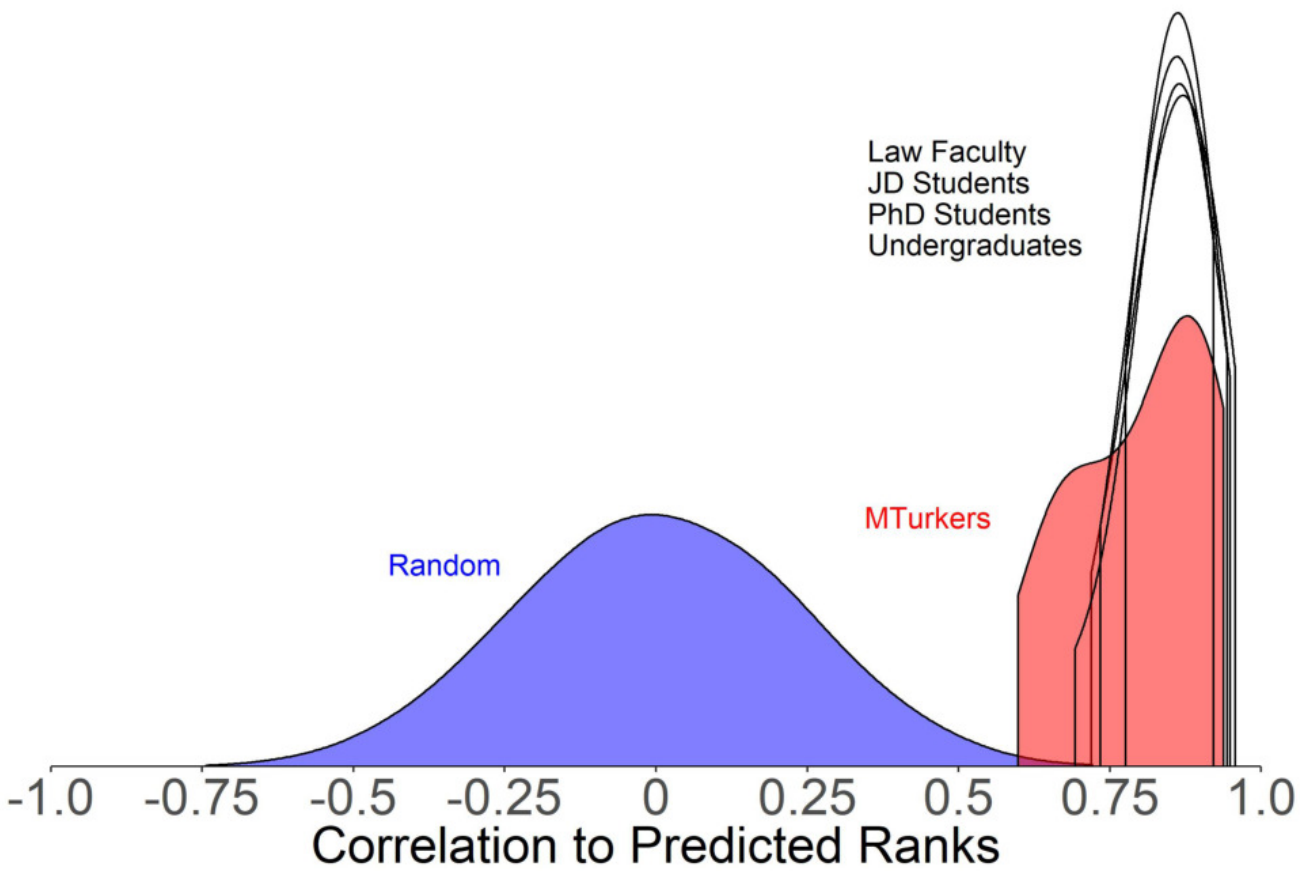

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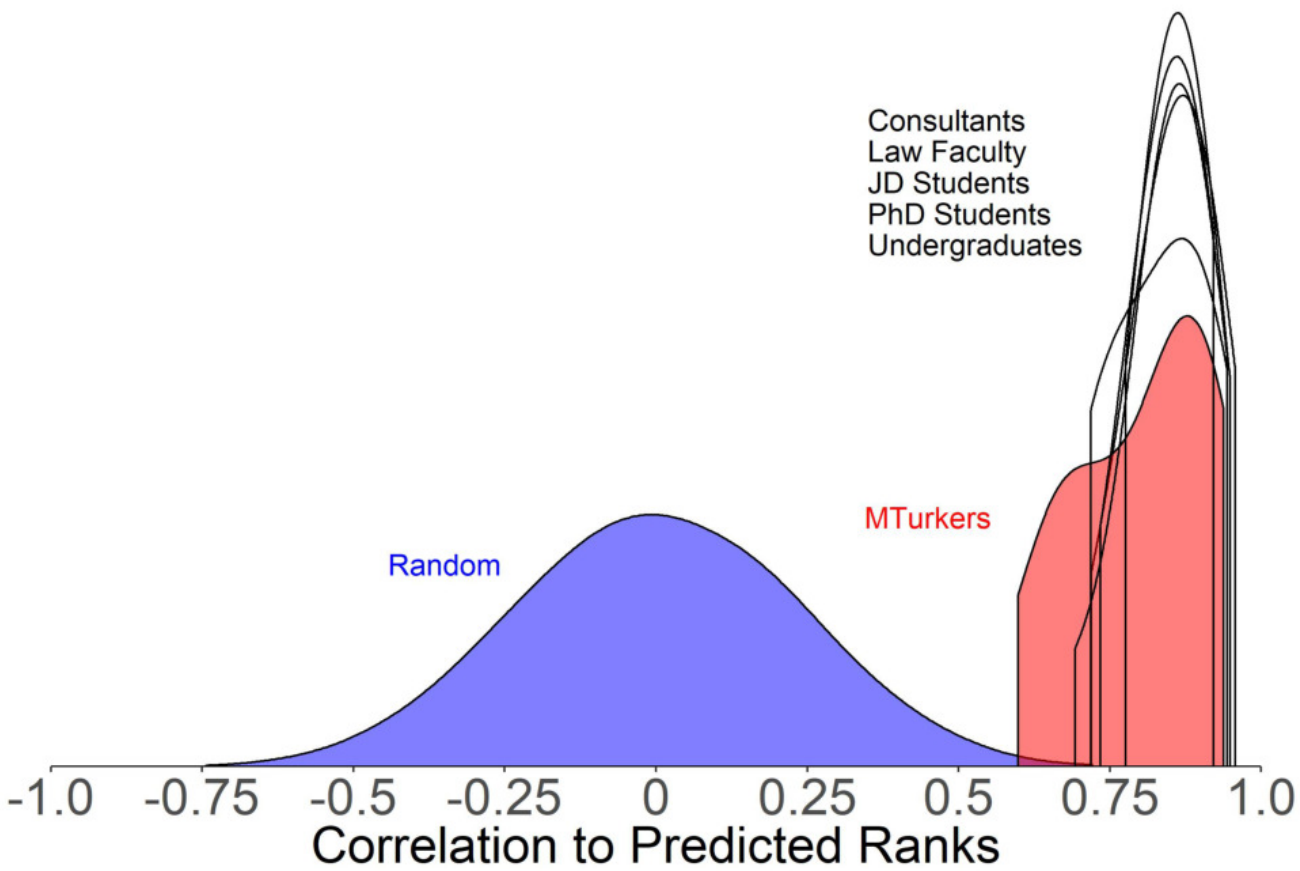

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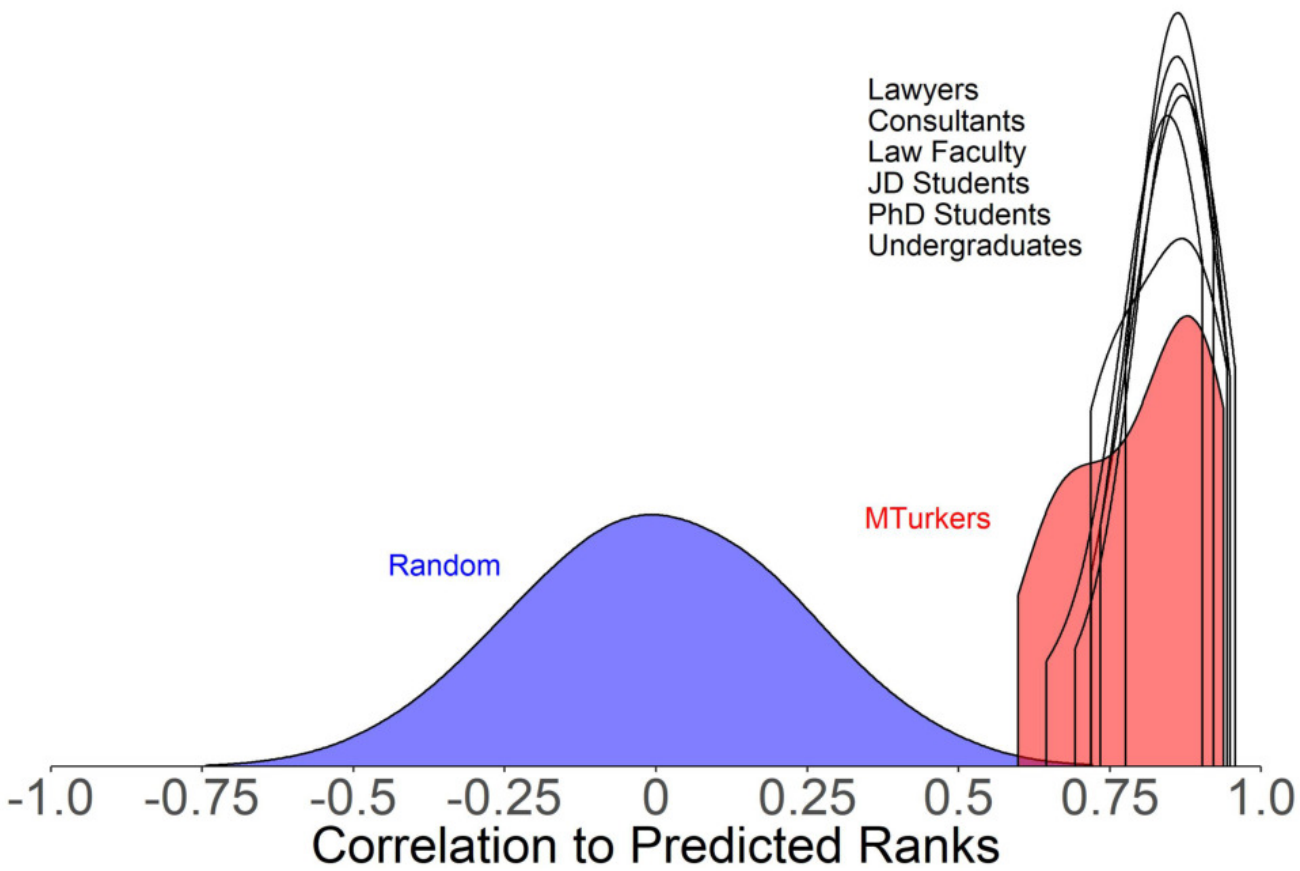

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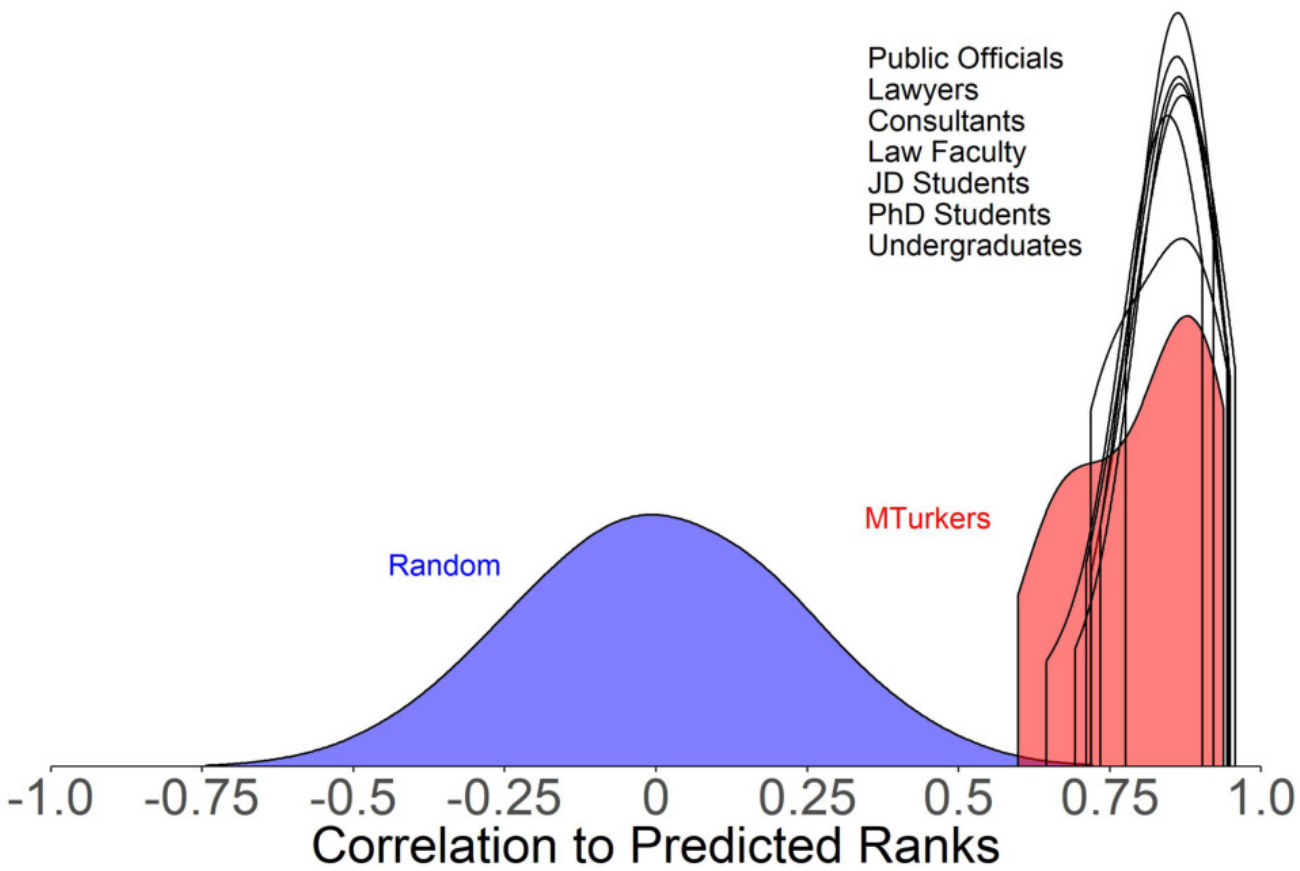

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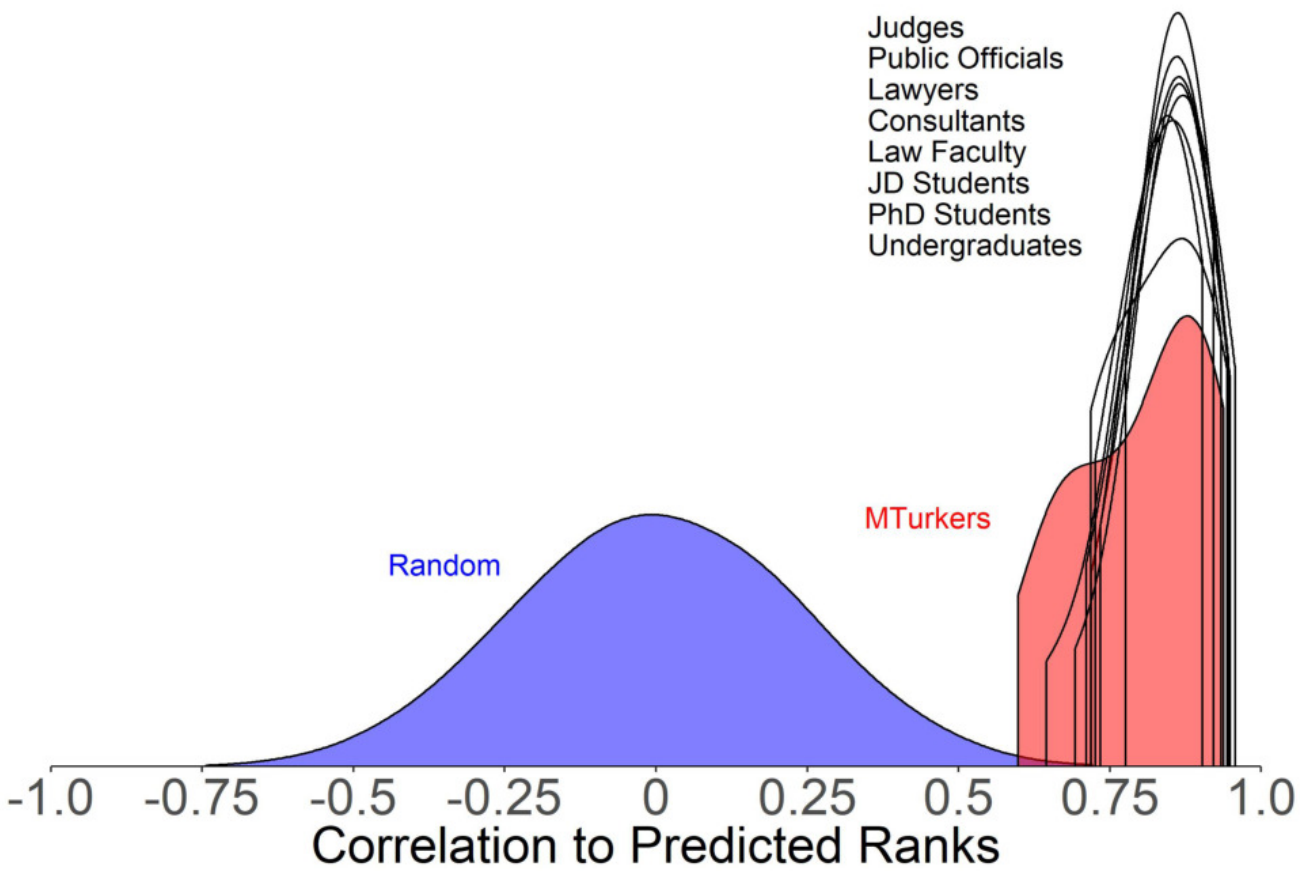

What do you think?

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Our measure:	COMPACT	noncompact	noncompact	COMPACT
Existing measure:	COMPACT	noncompact	COMPACT	noncompact

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Sam
Adam
Jason
Rick
Zachariah
Sandra
Amanda
Ari
Hannah
Indraneel
$48

Mean income:

Quantity of Interest

Theories of Inference: Statistics vs. CS

Population	Sample
⋮	✗
Sam	✓
Adam	✓
Jason	✓
Rick	✓
Zachariah	✓
Sandra	✓
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Hannah	✓
Indraneel	✓
$48	

Mean income:

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Theories of Inference: Statistics vs. CS

Population	Sample	$
⋮	✗	?
Sam	✓	122
Adam	✓	76
Jason	✓	145
Rick	✓	96
Zachariah	✓	86
Sandra	✓	127
Amanda	✓	72
Ari	✓	132
Hannah	✓	95
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$48		$108

Mean income:

Classical Inference

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Usually no direct relevance

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Noise & Censoring

Mean income: $48 — Classical Inference — $108

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Population	Sample	$	+Privacy	=dp$
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Adam	✓	76		103
Jason	✓	145	Noise & Censoring	75
Rick	✓	96		113
Zachariah	✓	86		125
Sandra	✓	127		97
Amanda	✓	72		101
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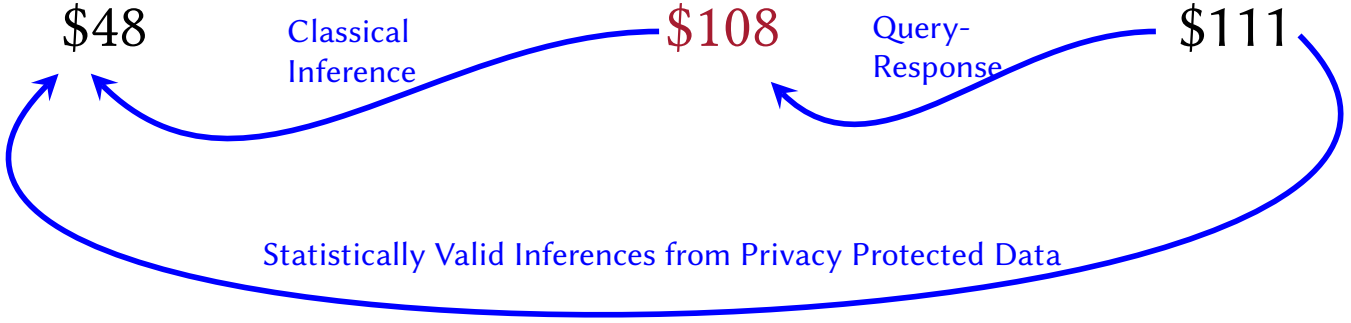

Analysis of Differentially Private Data (Data + Noise)

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  - The larger CIs: too large for my QOI

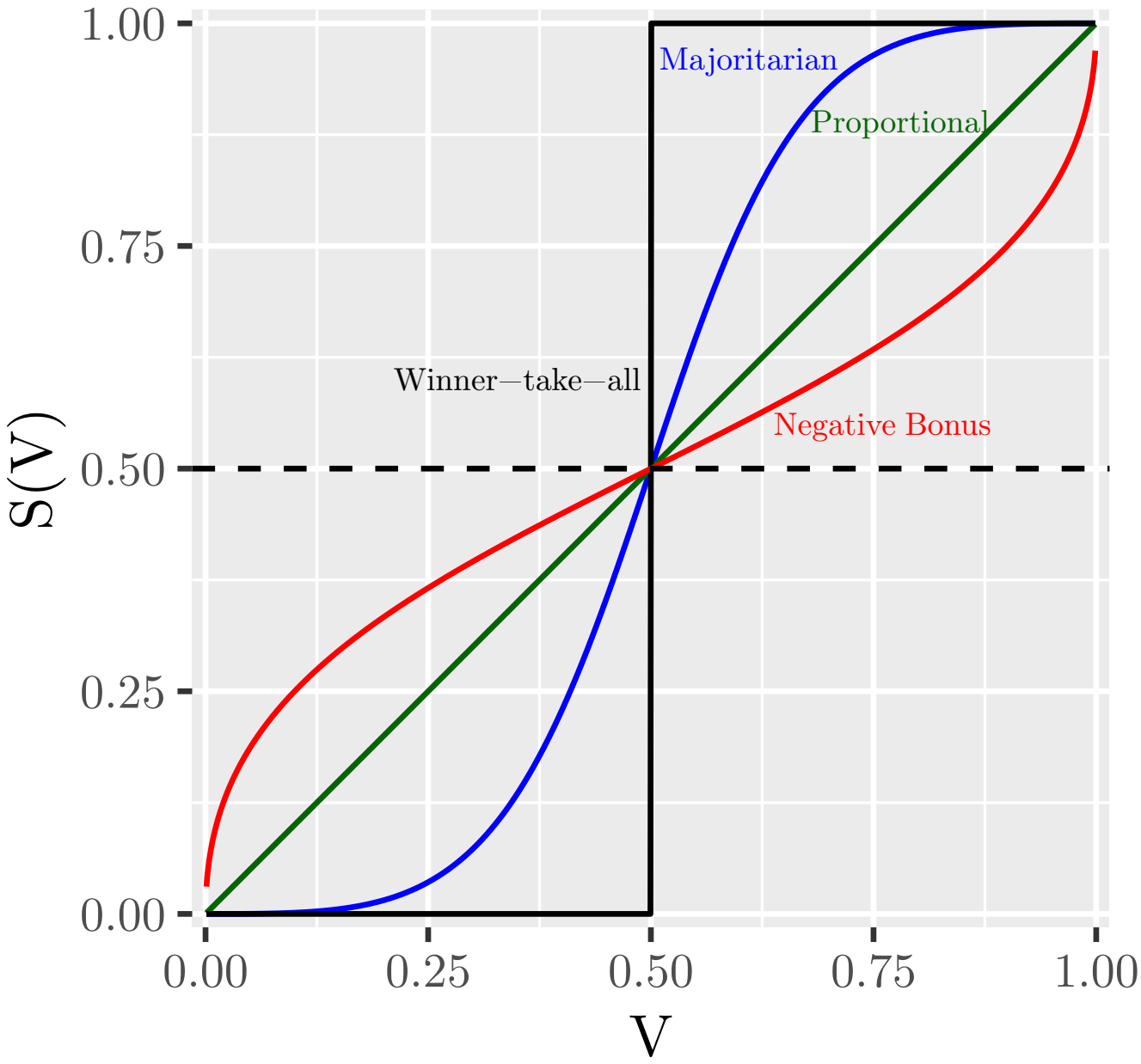
Principles

Defining and Measuring Compactness

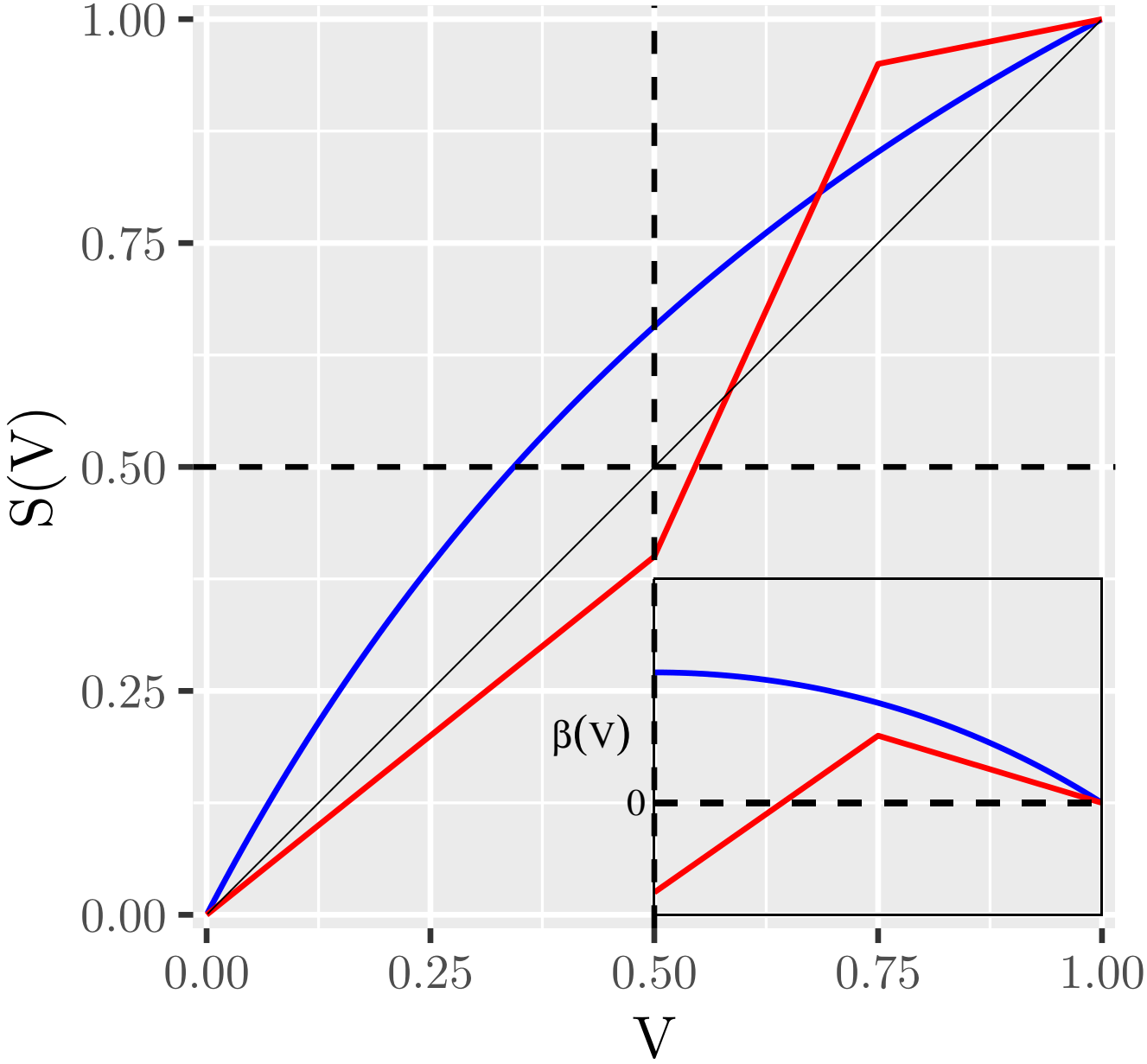
Differential Privacy

Fairness in Redistricting

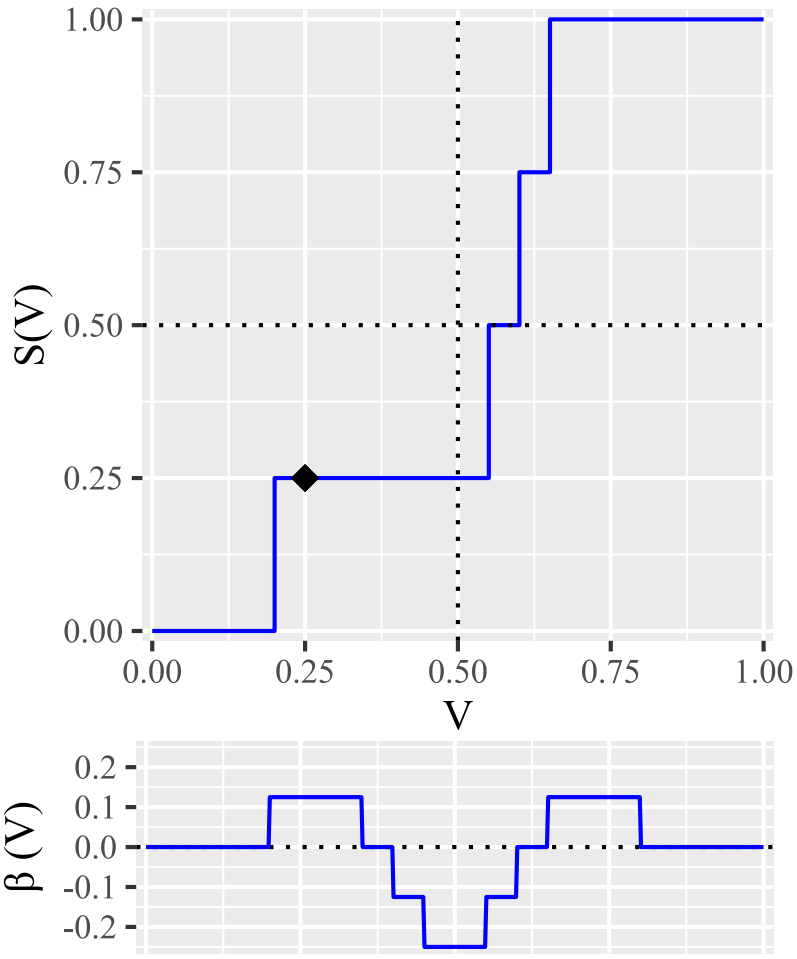
Symmetric ("Fair") Seats-Votes Curves

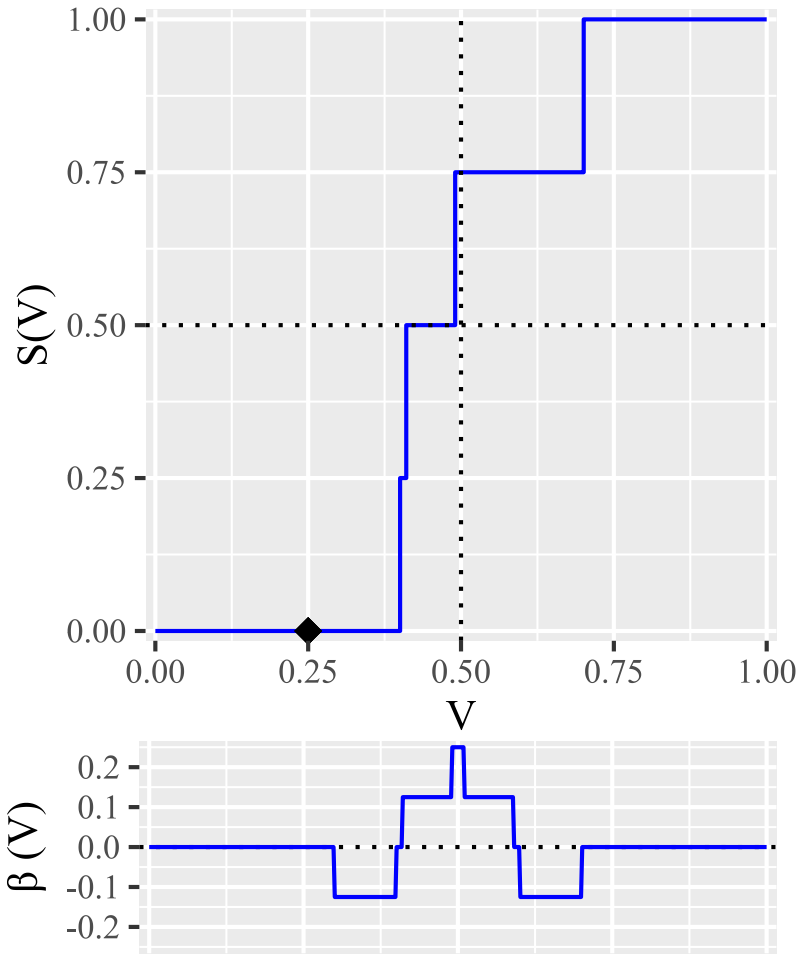


Asymmetric ("Biased") Seats-Votes Curves



Limited QOIs Revealed by Full Seats-Votes Curves





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For more information
GaryKing.org